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A contribution to the resolution of structural dynamics problems using frequency response function matrix

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Abstract

In structural dynamics, several problems are solved using formulations using frequency response function matrices. This work focuses on the exploitation and evaluation of these matrices. A technique of structural modifications, based on knowledge of the introduced modifications and the frequency response functions relating to the original structure, will first be described. Next, we will interest in the evaluation of the used flexibility matrices. These latter can be either calculated from a mathematical model or derived from experimental observations. In practice, only a limited number of columns of the dynamic flexibility matrix can be measured. A technique for completing this matrix is proposed after having described classical techniques. The idea is combined with a procedure which permits to choose, for numerical tests, an optimal placement of excitations. The proposed formulations are validated by a numerical example; and the effects of choice of number and positions of exciters, and the effect of damping on the results are discussed.

1. Introduction

To optimize calculations in structural dynamics, we are often confronted to solve formulations using Frequency Response Functions (FRF) matrices, like dynamic sub-structuring or structural modifications problems [1, 2]. In practice, this resolution is based on the knowledge of the frequency response function (FRF) matrix $\boldsymbol{H}(\omega)$. This matrix can be estimated either from an analytical or numerical simulation model, similar to the real model, or from experimental data.

In the experimental case, the matrix $\boldsymbol{H}(\omega)$, at each frequency in the analyzed band, is often evaluated either by reconstruction from the identified eigensolutions of the system, which requires a previous modal identification [3], or by direct measurement of all its independent elements. This last situation is rarely applied, because it's not economical, therefore only a very limited number of columns of the dynamic flexibility matrix can be measured, and consequently the other columns must be estimated.

In this work, we first develop a technique of structural modifications based on the knowledge of the frequency response functions relative to the original structure and the introduced modifications. Next, we propose, after having exposed conventional techniques for estimating the dynamic flexibility matrix, a technique which allows to evaluate the complete matrix without using the modal identification. A similar principle has already been proposed in the references [4, 5] and the idea is extended and combined with a procedure which permits to choose, for numerical simulations, an optimal placement of excitations [6].

A numerical simulation example will be proposed to validate the proposed formulations, and to discuss the effects of choice of number and positions of exciters, used to measure flexibility matrices, and the effect of damping on the quality of the evaluation.

2. Structural modification problems via transfer functions

2.1. General formulation

The modified structure can be represented by an assembly of two subsystems: the initial structure and an additional system constituted by the introduced modifications.

The equation representing the particular solution of the structure in its initial state, under a harmonic excitation, is expressed in matrix form as

$$\boldsymbol{z}(\omega) = \boldsymbol{H}(\omega)\boldsymbol{f} \tag{1}$$

Where $\boldsymbol{H}(\omega) \in \boldsymbol{C}^{c,c}$ is the symmetric FRF matrix of the initial structure (abbrev. I.S.), at the frequency ω, c is the number of pickup degrees-of-freedom (DOF) and $\boldsymbol{z}(\omega),\ \boldsymbol{f} \in \boldsymbol{C}^{c,1}$ represent the response vectors and external force, respectively.
To reduce the writing, we omit the argument ω. The above equation is partitioned in the form:

$$\begin{pmatrix} z_i \\ z_a \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_{ii} & \boldsymbol{H}_{ia} \\ \boldsymbol{H}_{ai} & \boldsymbol{H}_{aa} \end{pmatrix} \begin{pmatrix} \boldsymbol{f}_i \\ \boldsymbol{f}_a \end{pmatrix} \tag{2}$$

Where a denotes the DOF affected by the modification, and i denotes the other DOF.
The additional system, constituted by some known parametric modifications that not alter the order of the system, is represented by the dynamic stiffness matrix:

$$\Delta\boldsymbol{Z} = \left\lfloor \Delta\boldsymbol{K}_{\boldsymbol{aa}} + j\omega\Delta\boldsymbol{B}_{\boldsymbol{aa}} - \omega^2\Delta\boldsymbol{M}_{\boldsymbol{aa}} \right\rfloor \in \boldsymbol{C}^{a,a} \tag{3}$$

where $\Delta\boldsymbol{K}_{aa}$, $\Delta\boldsymbol{M}_{aa}$, $\Delta\boldsymbol{B}_{aa} \in \boldsymbol{R}^{a;a}$ are the symmetric stiffness, mass and damping matrices of the structural modification, respectively.
The linking forces vector $\tilde{\boldsymbol{f}}_{al}$ exerted by the I.S. on the additional system can be written (after condensation on the DOF of connection with the I.S.):

$$\tilde{\boldsymbol{f}}_{al} = \Delta\boldsymbol{Z}_{a,a}\tilde{z}_a \in \boldsymbol{C}^{a,1} \tag{4}$$

Where $\tilde{z}_a$ is the displacement vector of the additional system at the connection points with the I.S.
The flexibility relation of the modified structure (abbev. M.S.) is written:

$$\begin{pmatrix} \hat{z}_i \\ \hat{z}_a \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_{ii} & \boldsymbol{H}_{ia} \\ \boldsymbol{H}_{ai} & \boldsymbol{H}_{aa} \end{pmatrix} \begin{pmatrix} \hat{\boldsymbol{f}}_i \\ \hat{\boldsymbol{f}}_a \end{pmatrix} \tag{5}$$

with : $\hat{\boldsymbol{f}} = \boldsymbol{f}_{\boldsymbol{i}} \in \boldsymbol{C}^{c-a,1}$ and $\hat{\boldsymbol{f}}_a = \boldsymbol{f}_{\boldsymbol{a}} + \boldsymbol{f}_{\boldsymbol{al}} \in \boldsymbol{C}^{a,1}$, $\boldsymbol{f}_{\boldsymbol{al}}$ is the linking forces vector exerted by the additional system on the DOF of type "a".
The connection conditions are:

$$\tilde{z}_a = \hat{z}_a\ ;\ \ \tilde{\boldsymbol{f}}_{al} + \boldsymbol{f}_{\boldsymbol{al}} = 0 \tag{6}$$

After using equations (4) and (6), equation (5) can be written as:

$$\begin{pmatrix} \hat{z}_i \\ \hat{z}_a \end{pmatrix} = \begin{pmatrix} \boldsymbol{H}_{ii} - \boldsymbol{H}_{ia}\Delta\boldsymbol{Z}_{aa}\boldsymbol{W}\boldsymbol{H}_{ai} & \boldsymbol{H}_{ia}(\boldsymbol{I}_{\boldsymbol{a}} - \Delta\boldsymbol{Z}_{aa}\boldsymbol{W}\boldsymbol{H}_{aa}) \\ \boldsymbol{W}\boldsymbol{H}_{ai} & \boldsymbol{W}\boldsymbol{H}_{aa} \end{pmatrix} \begin{pmatrix} \boldsymbol{f}_i \\ \boldsymbol{f}_a \end{pmatrix} \tag{7}$$

with : $\boldsymbol{W} = \left[\boldsymbol{I}_a + \boldsymbol{H}_{aa}\Delta\boldsymbol{Z}_{aa}\right]^{-1}$.
Using equation (7), one can express the forced responses of the M.S., without recourse to an exact but costly reanalysis, by using only the dynamic flexibility matrix of the I.S. and the dynamic stiffness matrix of the introduced modification. The modal parameters of the M.S. are then accessed by applying a modal identification method on the previous frequency responses. In order to evaluate the FRF of the M.S. from (7), we have to determinate the matrix $\boldsymbol{W}$ at each frequency ω. This evaluation cost depends of the number a of DOFs affected by structural modifications.

2.2. Case of connecting DOFs to ground
For problems of attached DOFs to ground, in the simplest case, we choose for perturbation matrices $\Delta\boldsymbol{M}_{aa} = 0$ and $\Delta\boldsymbol{K}_{aa}$ as a diagonal matrix with very big diagonal elements. Then, the stiffness perturbation connects quasi-rigidly the a DOFs to the fixed reference. Equation (7) reduces to:

$$\hat{z}_i = \hat{\boldsymbol{H}}\boldsymbol{f}_{\boldsymbol{i}} \tag{8}$$

where: $\hat{\boldsymbol{H}} = \boldsymbol{H}_{ii} - \boldsymbol{H}_{ia}\Delta\boldsymbol{K}_{aa}\boldsymbol{W}\boldsymbol{H}_{ai}$ and $\boldsymbol{W} = \left[\boldsymbol{I}_a + \boldsymbol{H}_{aa}\Delta\boldsymbol{K}_{aa}\right]^{-1}$.

If we take ΔK_{aa} in the following form:

$\Delta K_{aa} = kI_a$, k is a positive scalar and I_a the unit matrix of order a.

the matrices W and $\hat{H}$ become :

$$W = \frac{1}{k} \times \left[\frac{1}{k} I_a + H_{aa} \right]^{-1} ; \; \hat{H} = H_{ii} - H_{ia} \left[\frac{1}{k} I_a + H_{aa} \right]^{-1} H_{ai} .$$

and for k tending to infinity, $\hat{H}$ is written:

$$\hat{H} = H_{\text{ii}} - H_{\text{ia}} \left[H_{\text{aa}} \right]^{-1} H_{\text{ai}} \tag{9}$$

In this formulation, the introduction of structural modifications is avoided, but we are always confronted with the inversion of the sub-matrix H_{aa} of order equal to the number of fixed DOFs. One can find the same formulation that (9), but established with a different way, by using (5) and imposing the constraint $\hat{z}_a = 0$.

3. Evaluation of the FRF matrix

For solving structural modifications problems defined in (7), for example, we must know the dynamic flexibility matrix of the I.S. which can be estimated in various ways.

3.1. Estimation from an updated finite element model

In the dynamics of mechanical structures, a continuous system is often discretized and represented by models consisting of a limited number n of DOFs [7, 8]. A first way to determinate the FRF matrix $H(\omega) \in C^{n,n}$, at a frequency ω, is by a calculation from an available finite element model. If we note M, B and K, respectively the mass, damping and stiffness matrices of the structure, the FRF matrix is then calculated by the following relation:

$$H(\omega) = (K + j\omega B - \omega^2 M)^{-1} \tag{10}$$

This can be a computationally very intensive calculation in the case of component models with a large number of DOFs and/or a wide excitation frequency range. After all, the dynamic stiffness matrix has to be inverted for every discrete frequency in the frequency range of interest.

3.2. Estimation using experimental measurements

When data are resulting from experimental measurements, we are often constrained to operate with a reduced sub matrix $H_{cc} \in C^{c,c}$ where: c ($c << n$) represents the limited number of sensors which have been optimally placed on the tested structure [9, 10].

The elements of $H_{cc}(\omega)$ are generally evaluated either by reconstruction using identified eigensolutions, or by direct measurement of its $c\times(c+1)/2$ independent elements.

3.2.1 Reconstruction using identified eigensolutions

A second way to determine the FRFs of a damped structure is by using an FRF synthesis based on a finite number of eigenvectors and eigenfrequencies of the structure. If we consider an n *DOF* structure whose behaviour is represented on the basis of its $2n$ complex modes, the relationship between the synthesized FRF matrix $H(\omega)$ and eigenvectors is expressed by

$$H(\omega) = \Psi (j\omega I - S)^{-1} \Psi^T + \overline{\Psi} (j\omega I - \overline{S})^{-1} \overline{\Psi}^T \tag{11}$$

Where $\Psi \in C^{n,n}$, $S = Diag[s_i] \in C^{n,n}$ represent respectively the modal and spectral matrices of the structure and $\overline{\Psi}$, $\overline{S}$ are respectively the conjugate matrices of Ψ and S.

Usually, the number m of identified modes is less than the total number n of DOFs ($m << n$). In the given frequency band containing the modes measured, one can express $H(\omega)$ as:

$$H(\omega) = H^d(\omega) + H^r(\omega) \tag{12}$$

Where $H^d(\omega),\ H^r(\omega) \in C^{n,n}$ represent the contributions of the eigenmodes inside and outside the observed frequency band, respectively. The matrix $H^d(\omega)$ is defined by :

$$H^d(\omega) = \Psi_1(j\omega I_m - S_1)^{-1}\Psi_1^T + \overline{\Psi}_1(j\omega I_m - \overline{S}_1)^{-1}\overline{\Psi}_1^T \tag{13}$$

Where: $\Psi_1 \in C^{n,m}$, $S_1 \in C^{m,m}$ are respectively modal and spectral sub-matrices corresponding to the m identified eigenmodes.
In order to compensate partially the contribution of the ($n - m$) unidentified modes [14], in the observed band, the part $H^r(\omega)$ of $H(\omega)$ is frequently approximated by their static contribution:

$$H^r(\omega) \cong H^r(0) = H(0) - H^d(0) \tag{14}$$

This compensation is important in the extern resonance zones of $H^d(\omega)$, where the static contributions to the response of the modes which have not been measured are significant.
Like already mentioned above, we will use only the sub-matrix $H_{cc}(\omega) \in C^{c,c}$ ($m < c < n$) of $H(\omega)$ relative to the c pickup DOFs. The matrix $H_{cc}(\omega)$ is defined by:

$$H_{cc}(\omega) = H_{cc}^{d}(\omega) + H_{cc}^{r}(\omega) \tag{15}$$

Where:

$$H_{cc}^d(\omega) = \Psi_{1c}(j\omega I_m - S_1)^{-1}\Psi_{1c}^T + \overline{\Psi}_{1c}(j\omega I_m - \overline{S}_1)^{-1}\overline{\Psi}_{1c}^T \tag{16}$$

$$H_{cc}^r(\omega) \cong H_{cc}^r(0) = H(0) - H_{cc}^d(0)$$

$\Psi_{1c} \in C^{n,m}$ ($m < c$) is the modal sub-matrix of Ψ_1 at the c observed DOFs.

To estimate $H_{cc}(\omega)$, we need to identify the matrices Ψ_{1c} , S_1 and $H_{cc}^r(\omega) \in C^{c,c}$. For that, only p ($p < c$) columns or lines of $H_{cc}(\omega)$ are sufficient [3], these ones are measured by applying linearly independent excitations in the observed frequency band. Thus, equations (15) and (16) allow the matrix $H_{cc}(\omega)$ to be evaluated from a much small number of observed columns p among the c columns. Several modal identification methods have been developed for this purpose. One can see, for example, reference [3]. In order to avoid a costly modal identification of the three matrices Ψ_{1c}, S_1 and $H_{cc}^r(\omega)$ an alternative method is proposed, it is based on the direct exploitation of a knowledge sub-matrix $H_1(\omega) \in C^{c,p}$ of $H_{cc}(\omega)$.

3.2.2 Direct evaluation of the FRF matrices

In this purpose, the contributions of all the structural modes are taken into account. The entire knowledge of $H_{cc}(\omega)$ requires c sensors and c excitations. Usually, for economic reasons, only a limited number p of linearly independent excitation configurations is available.
Problem: Knowing p ($p < c$) columns from $H_{cc}(\omega)$ denoted by the sub-matrix $H_1(\omega) \in C^{c,p}$, we have to estimate (at the best) the $c - p$ remaining columns without performing a modal identification.
In the following, a technique which contributes to the resolution of this problem is described. As references to similar method we can see [4, 5].
To precise the unknowns of the problem, the FRF matrix $H_{cc}(\omega)$ is partitioned into sub matrices as:

$$H_{cc} = \begin{pmatrix} H_1 & H_2 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \tag{17}$$

Where: $H_1 \in C^{c,p}$ is the known part of H_{cc}, $H_{11} \in C^{p,p}$ a square sub-matrix of H_1 and $H_2 \in C^{c,c-p}$ is the unknown part of H_{cc}.
We only consider cases where the FRF matrix $H_{cc}(\omega)$ is symmetric:

$$H_{12} = H_{21}^T,\ H_{11} = H_{11}^T,\ H_{22} = H_{22}^T.$$

In this case, the number of unknown elements of the rectangular matrix $\boldsymbol{H}_2$ is contained in the square matrix $\boldsymbol{H}_{22}$. Thus the total number of unknown elements is reduced to $c\times(c-p+1)/2$.

a) Evaluation by using spectral factorization of the square measured sub-matrix $\boldsymbol{H}_{11}$

The eigenvalues γ_i and eigenvectors φ_i (i=1, ...,p) of the matrix $\boldsymbol{H}_{11}$ are defined by the eigenvalues problem [12]:

$$\left(\boldsymbol{H}_{11}-\gamma_i\boldsymbol{I}_p\right)\varphi_i=0,\ i=1,\ \ldots,p \tag{18}$$

One can then write the complex symmetric matrix $\boldsymbol{H}_{11}$ in the form:

$$\boldsymbol{H}_{11}=\boldsymbol{\Phi}_{11}\boldsymbol{\Gamma}\boldsymbol{\Phi}_{11}^T \tag{19}$$

Where: $\boldsymbol{\Gamma},\boldsymbol{\Phi}_{11}\in\boldsymbol{C}^{p,p}$ are the diagonal matrix of eigenvalues and the modal matrix of eigenvectors of $\boldsymbol{H}_{11}$, respectively. These eigenvectors are normed such that:

$$\boldsymbol{\Phi}_{11}\boldsymbol{\Phi}_{11}^T=\boldsymbol{\Phi}_{11}^T\boldsymbol{\Phi}_{11}=\boldsymbol{I}_p \tag{20}$$

The factorization (19) is valid for the matrices with distinct eigenvalues and possibly for the matrices with multiple eigenvalues.

To estimation the whole FRF matrix, let us look for the matrix $\boldsymbol{\Phi}_{21}\in\boldsymbol{C}^{c-p,p}$ such that:

$$\begin{pmatrix}\boldsymbol{H}_{11}\\ \boldsymbol{H}_{21}\end{pmatrix}=\boldsymbol{\Phi}\boldsymbol{\Gamma}\boldsymbol{\Phi}_{11}^T,\ \boldsymbol{\Phi}=\begin{pmatrix}\boldsymbol{\Phi}_{11}\\ \boldsymbol{\Phi}_{21}\end{pmatrix} \tag{21}$$

It can be deduced from this that:

$$\boldsymbol{\Phi}_{21}=\boldsymbol{H}_{21}\boldsymbol{\Phi}_{11}\boldsymbol{\Gamma}^{-1} \tag{22}$$

Thus the sub-matrix $\boldsymbol{H}_{22}$ is approached by:

$$\boldsymbol{H}_{22}=\boldsymbol{\Phi}_{21}\boldsymbol{\Gamma}\boldsymbol{\Phi}_{21}^T$$

Consequently the complete FRF matrix $\boldsymbol{H}_{cc}$ is approximated by the matrix:

$$\widetilde{\boldsymbol{H}}_{cc}=\begin{pmatrix}\boldsymbol{\Phi}_{11}\\ \boldsymbol{\Phi}_{21}\end{pmatrix}\boldsymbol{\Gamma}\begin{pmatrix}\boldsymbol{\Phi}_{11}^T & \boldsymbol{\Phi}_{21}^T\end{pmatrix} \tag{23}$$

Reviews

- In the case where some diagonal elements γ_i of the diagonal matrix $\boldsymbol{\Gamma}$ are very low values, it is obvious that the calculation of $\boldsymbol{\Phi}$ can influence the evaluation quality.
- Generally the eigenvalues of a matrix do not give precise information about its rank. If it is desired to control the rank of the matrix $\boldsymbol{H}_{11}$, it is preferable to use singular value decomposition [5, 13].

b) Complementary formulation

Numerical simulations show that, with the previous formulation, the quality of evaluation in the neighborhoods of anti-resonance frequencies (low amplitude regions) is poor, but in the regions of resonance frequencies the quality is practically perfect. In this case, for improving the estimation in the low amplitude regions, one can exploit an idea already proposed in [5], where the flexibility matrix can be expressed, at each frequency ω of the analyzed band, by a linear combination of the dynamic flexibilities at resonance frequencies. The steps to follow are described below:

(1) Estimate approximately, from the measured FRF rectangular sub-matrix $\boldsymbol{H}_1$, the m resonance frequencies ω_i (i = 1, …, m) in the analysis band;

(2) Calculate $\boldsymbol{H}_{cc}(\omega_i)$, for i = 1, …, m, by eq. (23);

(3) Using the least squares process, find the coefficients $x_i(\omega)\in\boldsymbol{C}$, i = 1, …, m which can verify:

$$H_1(\omega) = \sum_{i=1}^{m} x_i(\omega) H_1(\omega_i) \quad (24)$$

(4) In end, by analogy with (24), calculate $H_{cc}(\omega)$, at each frequency ω of the analyzed band, by using the results of preceding steps, which give:

$$H_{cc}(\omega) = \sum_{i=1}^{m} x_i(\omega) H_{cc}(\omega_i) \quad (25)$$

It is evident that the quality of evaluation of the FRF matrix depends particularly on the positions and the number of sensors and exciters, the structure damping and the spectral density.

c) Choice the best columns of H_1

To proof that the estimation of H_{cc} depends on choice of the excitations, one can exploit the methods presented in references [5, 6, 7].

For numerical simulations presented in section 4, we will choose the rectangular sub-matrix H_1, containing the p known columns of H_{cc}, by using the combinatorial method presented by Majed in [6]. This technique is constructed from an available finite element model, and permits to have the best positions of p excitations between g DOFs possibly excitable (i.e. it permits to choose p linearly independent columns of the FRF matrix among g columns). The technique consists in generating all possible combinations of potential p DOFs among the picked DOFs; a criterion for the final selection of the best combinations is then applied among those obtained previously.

4. Results and discussion

To illustrate the procedure relative to the evaluation of the FRF matrix, one considers the 2D frame represented in Figure 1. The structure is modeled by using a finite element code. This model is discretized into 22 finite beam elements; it contains 20 unconstrained nodes with 3 DOFs per node. The frame has the following physical and geometrical characteristics:

- Beam section $S = 1.392\times10^{-4}\ N/m^2$; moment of inertia $I = 2.673\times10^{-10}\ m^4$;
- Young's modulus $= 2.1\times10^{11}\ N/m^2$; density $\rho = 7800\ kg/m^3$;
- $L_1 = 0.6\ m$; $L_2 = 0.45\ m$; $L_3 = 0.36\ m$.

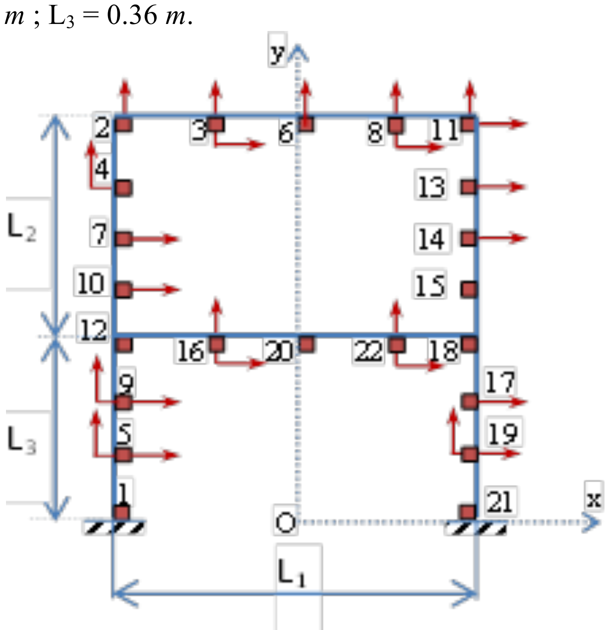

Figure 1: 2D frame (→ Positions of c = 24 sensors)

A proportional damping ($BM^{-1}K = KM^{-1}B$) is introduced and the "exact" FRF matrix $H(\omega)$ is calculated, at each frequency ω in the analyzed band, by using the eigenmodes of the dissipative structure. We note $a_i = |Re(s_i)| / Im(s_i)$ the i^{th} modal damping factor; $s_i = -a_i\omega_i + j\omega_i$ is the i^{th} eigenvalue of the structure. The frequency band under consideration [0, 200 Hz] contains the first 9 eigenfrequencies of the structure (see Table 1).

Table 1: Reference values of the first 10 eigenfrequencies of the frame

Mode number	1	2	3	4	5	6	7	8	9	10
Frequencies (Hz)	8.76	29.37	43.76	56.21	96.04	102.57	147.40	175.11	179.60	206.89

We suppose that we know p columns of the matrix $\boldsymbol{H}_{cc}(\omega)$ and we look for the remaining $c-p$ columns. A total of c = 24 pickups DOFs are arbitrarily chosen.

In order to justify that the quality of the evaluation depends on the positions and the number of the excitations, and the damping of the structure, we consider the following cases.

(a) Choice of p=4 arbitrarily selected excitation points, with a modal damping factor a_i = 0.01, i = 1, 2, …

The remaining 20 columns of the matrix $\boldsymbol{H}_{cc}(\omega)$ are determined on the basis of the 4 measured columns, yielding a total of 138 unknown elements if symmetry is taken into account. In *Fig. 2,* the evolution of the amplitude of one unknown element H_{ij} is plotted as a function of ω and compared with the exact values. This curve shows that an arbitrary choice of the locations of 4 exciters leads to mediocre evaluation quality in some regions of the frequency band. The expansion is nearly perfect in the neighborhood of the resonances but remains poor in the low amplitude regions, especially near the anti-resonances. A certain number of parasitic resonances inadvertently appear in certain components of the estimated FRF matrix $\tilde{\boldsymbol{H}}_{cc}$. These peaks correspond to the resonance frequencies of the structure and though they do not appear on all the elements of the exact FRF matrix $\boldsymbol{H}_{cc}$, they can appear in the homologous expanded elements. These parasitic peaks can be attenuated by judiciously choosing the rank of $\boldsymbol{H}_{11}(\omega)$.

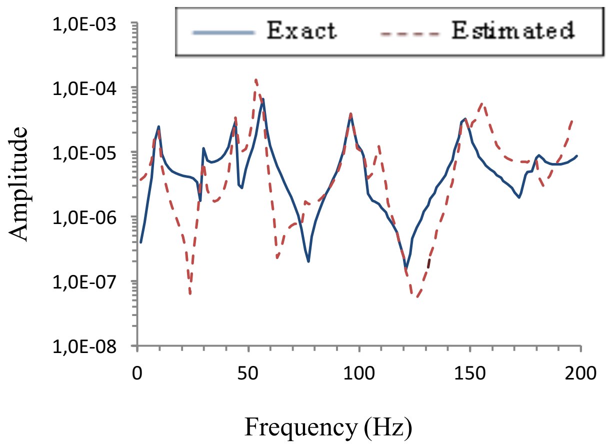

Figure 2: p=4 arbitrarily selected excitation points, with a modal damping factor a_i = 0.01

We also note that, for the element considered in the above figure, the estimated curve presents some spurious peaks which are not related to resonance frequencies of the structure, but to the fact that the matrix $\boldsymbol{H}_{11}$ may be ill-conditioned for some frequencies; it is thus during the factorization (19) that very small values will appear in the diagonal matrix $\boldsymbol{\Gamma}$, and which will become very large during its inversion. Therefore, it is equation (22) that can generate such peaks when the matrices considered are not of full rank. This means that the observed columns of the matrix $\boldsymbol{H}_{cc}$ must be selected intelligently.

(b) Choice of p=4 optimal selected excitation points, with a modal damping factor a_i = 0.01, i = 1, 2, …

This case is similar to the case *(a)* except that the p=4 exciters are here optimally chosen, by using the technique presented in [6]. The figure 3, show the evolution of the exact and estimated amplitudes of the same element H_{ij}. The results in this case are significantly better than the previous ones. But, there are still some regions where the estimation remains mediocre. We must note that choose of the best positions of excitations is an experimental problem. In the below case, we look what will happened if the number p of optimal exciters increases.

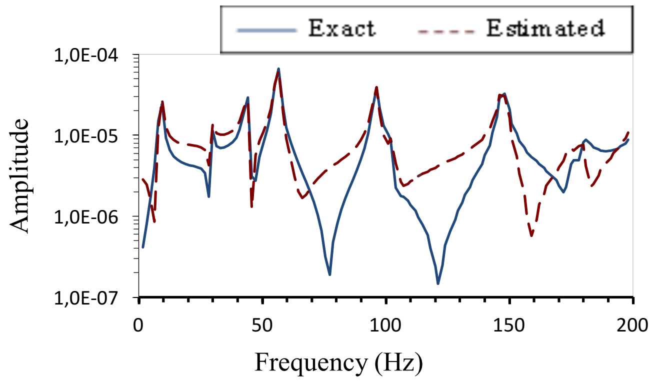

Figure 3: $p = 4$ optimal selected excitation points, with a modal damping factor $a_i = 0.01$

(c) Choice of $p = 8$ optimal selected excitation points, with a modal damping factor $a_i = 0.01$

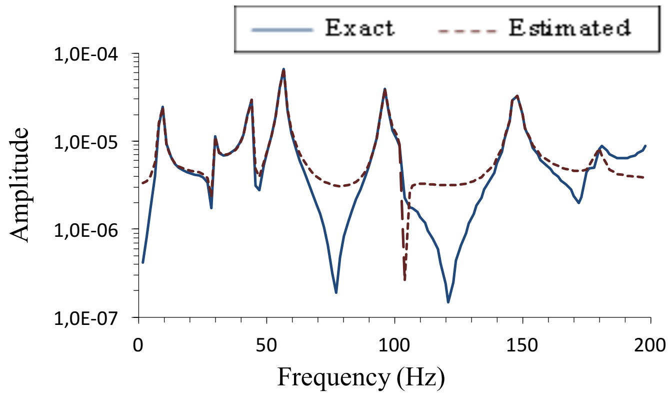

Figure 4: $p = 8$ optimal selected excitation points, with a modal damping factor $a_i = 0.01$

In this case we consider p=8 optimal selected excitation points, with the same modal damping factor $a_i = 0.01$, and we will study the same element H_{ij} as above. In Figure 4, we see that increasing p improves sensibly the quality of the results; especially near the resonances where the quality is perfect. Generally the results obtained are better than those of the cases *(a)* and *(b)*. Nevertheless, the estimation remains poor in the neighborhood of some anti-resonances but it's acceptable for those located in the frequency band [0, 60 Hz]. To improve the estimation in the neighborhood of anti-resonances, we use, after exploiting eq. (23) at the resonances frequencies, the results obtained by the complementary formulation eq. (25); Figure 5 illustrate this amelioration.

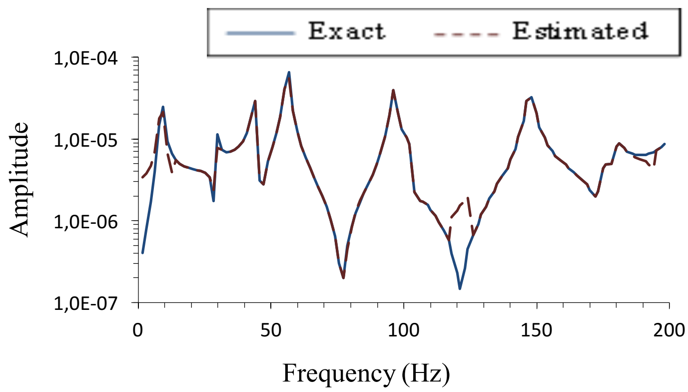

Figure 5: $p = 8$ optimal selected excitation points, with a modal damping factor $a_i = 0.01$, using eq. (25*)*

(d) Choice of p=8 selected excitation points, with a modal damping factor $a_i = 0.1$

The effect of damping is illustrated in Figure 6 for the same element of $\boldsymbol{H}_{cc}(\omega)$ as before except that the damping coefficients are now $a_i = 0.1$. In general, the results remain acceptable even if of the quality is lower in certain regions.

Note that, all the unknown elements are not same quality of estimation than the element H_{ij}. There are some ones that are better, some are the same quality and others are less better but with acceptable quality.

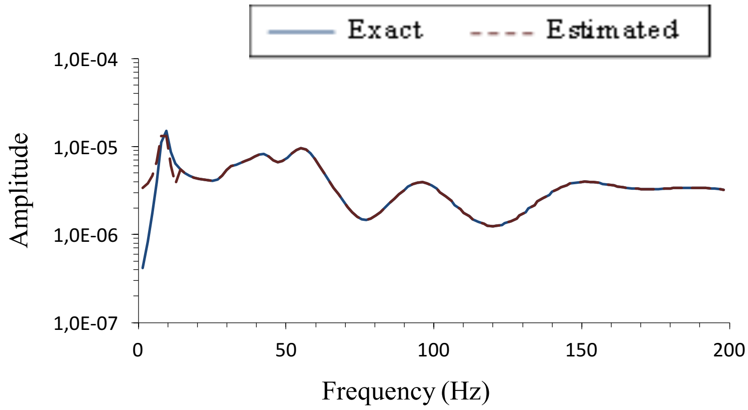

Figure 6: p = 8 optimal selected excitation points, with a modal damping factor a_i = 0.1

Conclusion

The objective was to contribute to the resolution of certain dynamic structures problems established from the FRF matrices. For this, in section 2, we have presented a formulation dealing with the reanalysis of modified structures problems, and discussed the case where some DOF can be rigidly connected to the ground. In general, the quality of the frequency responses of the modified structure depends on the quality of estimation of the flexibility matrices of the original structure. To do so, we have proposed, in paragraph 3.2.2, a method, based on a spectral decomposition of a square FRF sub-matrix; in some low amplitude regions, this method can be associated to the complementary formulation exposed in eq. (25) for improving the results.

Through the numerical simulations, we have seen that the estimation quality of the FRF matrix $\boldsymbol{H}_{cc}(\omega)$ depends on several factors. In the case where a degradation of the quality of the estimation is observed, even with a better choice of the positions of the exciters, an increase in the number of exciters can correct this defect. The increase in damping also makes it possible to improve the quality of the estimation and to attenuate the spurious peaks which appear in the spectrum.

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